Green’s function retrieval through cross-correlations in a two-dimensional complex reverberating medium

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Cross-correlations of ambient noise averaged at two receivers lead to the reconstruction of the two-point Green’s function, provided that the wave-field is uniform azimuthally, and also temporally and spatially uncorrelated. This condition depends on the spatial distribution of the sources and the presence of heterogeneities that act as uncorrelated secondary sources. This study aims to evaluate the relative contributions of source distribution and medium complexity in the two-point cross-correlations by means of numerical simulations and laboratory experiments in a finite-size reverberant two-dimensional (2D) plate. The experiments show that the fit between the cross-correlation and the 2D Green’s function depends strongly on the nature of the source used to excite the plate. A turbulent air-jet produces a spatially uncorrelated acoustic field that rapidly builds up the Green’s function. On the other hand, extracting the Green’s function from cross-correlations of point-like sources requires more realizations and long recordings to balance the effect of the most energetic first arrivals. When the Green’s function involves other arrivals than the direct wave, numerical simulations confirm the better Green’s function reconstruction with a spatially uniform source distribution than the typical contour-like source distribution surrounding the receivers that systematically gives rise to spurious phases. © 2014 Acoustical Society of America.

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I. INTRODUCTION

Over the last decade, the average of cross-correlations over long times and multiple sources that was initiated in helioseismology (Duvall et al., 1993) has become a powerful concept in seismic and acoustic imaging (Lobkis and Weaver, 2001; Shapiro and Campillo, 2004; Sabra et al., 2005; Roux et al., 2004). Focusing solely in geophysics, where impressive imaging results have been obtained since the first papers in 2004, cross-correlation observations managed to illuminate regions of the Earth that were previously invisible to tomographers (Mordret et al., 2013; Gao et al., 2011; Ritzwoller et al., 2007). As most ambient noise observed on Earth propagates in the form of short-period surface waves (with average period ~10 s) (Hillers et al., 2012), seismic-noise correlation is particularly relevant to crust-lithosphere imaging; however, recent studies suggest that the same techniques can be applied to lower-frequency signals sampling the entire mantle down to the core (Poli et al., 2012; Nishida, 2013).

The cross-correlation of the signal recorded at two stations consists of a so-called “causal” contribution that contains energy that travels from one reference station to another, and an “anti-causal” contribution that travels from the second station toward the reference station. If the noise sources are uniformly distributed or just sample the stationary-phase region (Snieder, 2004; Roux et al., 2005; Curtis et al., 2012), the causal and anti-causal parts of the cross-correlation are symmetric with respect to zero time (Fan and Snieder, 2009; Hillers et al., 2012), and the Green’s function is well approximated. As shown in recent studies in geophysics, non-uniformities in the geographic distribution of noise sources greatly complicate amplitude measurements, but still allow the estimation of the wave velocity, as the bias introduced in the time of flight is classically small (Froment et al., 2010; Tsai, 2010, 2009). The length of the cross-correlation time window is often regarded as alternative to the uniformity of the source spatial distribution, but the validity of this assumption and the trade-off between the two are still under study.

In the last 10 years, laboratory experiments have been conducted to explore the effects of source distribution and medium complexities. Using a solid reverberant body and diffuse thermal noise, Weaver and Lobkis (2001) wrote a seminal paper where the time-averaged cross-correlation between two receivers perfectly converges to the Green’s function. The same authors explored then the same idea with point-like ultrasonic sources (Lobkis and Weaver, 2001). They showed that the Green’s function extraction was more...
complicated, even if the cross-correlation was averaged over strongly reverberated signals obtained from sources at different locations. Moreover, small unexplained wavelets between the Green’s function and the cross-correlation function remained. A similar experiment was recently implemented with a non-reverberating solid that contains point scatterers (Mikesell et al., 2012). Using the signal generated by sources placed on a contour surrounding the target area, the Green’s function is reconstructed from the average of a large set of cross-correlations. The discrepancy between the averaged cross-correlation and the Green’s function, which was also found in this study, might suggest that in such complex media relaxing the source distribution requirements (Wapenaar and Fokkema, 2006) might not be a valid approach. Similar to these laboratory experiments, but at a larger scale, retrieval of reflections from seismic noise recorded at the surface in horizontally layered sediments was also polluted by the presence of unexpected arrivals (Draganov et al., 2007). Finally, at the global Earth scale, the reconstruction of reflected body waves using noise records from seismometers proved to be strongly dependent on the noise distribution and the reverberation generated by discontinuities/inhomogeneities within the medium (Poli et al., 2012; Boue et al., 2013).

The present paper deals with flexural waves in a reverberating two-dimensional (2D) plate (Larose et al., 2007), for which it was previously verified both numerically and experimentally that the direct path of the dispersive Green’s function can be satisfactorily approximated by stacked cross-correlations. Focusing now on the retrieval of reverberated phases, it is shown that the fit between the Green’s function and the cross-correlation depends on the nature and distribution of the contributing sources. In particular, numerical simulations first show that point sources uniformly distributed over the surface, even if limited in extent, efficiently participate in the Green function extraction when compared to point sources only located along a curve surrounding the receivers. Using laboratory experiments, such source configuration corresponds to the difference between striking a small portion of the plate with an air-jet coming from an air nozzle, and deploying a set of point sources along a line around the receiving points.

This study is organized in two parts. In the first part, acoustic numerical modeling is used in two dimensions to test the effects of the source distribution on the reconstruction of the direct wave and on one reflected wave. A large number of sources deployed in two different geometries are considered: i.e., (1) a dense distribution of sources along circles of different radii; and (2) an even distribution of sources on the surface. It is shown that non-physical arrivals are produced with a circular distribution. These spurious arrivals are well explained by simple geometrical considerations on the correlation terms between the direct and reflected paths. When the distribution of the sources is spatially uniform, the reconstruction of the Green’s function is improved. This illustrates the importance of source—receiver diversity in the framework of the cross-correlation theorem. In the second part, the problem of dispersive flexural waves that propagate in a finite plate is considered. The wave-field is made much more complex than in the previous case, because of the numerous reverberations on the plate edges. The numerical results indicate that for both the contour and surface distributions of the sources, the correlation is a better approximation of the Green’s function when long time series, including reverberations, are considered. Again, the surface distribution leads to better results in terms of reconstruction quality. Finally, laboratory experiments are performed in a plate in which different types of acoustic sources are compared. For the contour source distribution, the experiments confirm the presence of spurious arrivals in the cross-correlation, as observed with numerical simulation. To improve the spatial diversity, an air-jet was used as a spatially distributed incoherent source. The reconstruction is clearly improved compared to the stack of individual point sources.

A. Spatially versus azimuthally uniform source distribution

The theory of diffuse wave-field interferometry is based on the finding that by (1) cross-correlation of the signal generated by a point source at two receivers, (2) iteration over a sufficiently large set of more or less uniformly distributed sources, and (3) stacking (or averaging) of the cross-correlations, the resulting trace approximates the Green’s function associated with the locations of the two receivers (Sabra et al., 2005; Wapenaar, 2004; Boschi et al., 2013). Mathematically, this is expressed by the following expression at frequency \( \omega \) and valid for a 2D medium:

\[
G_{12} - G_{21}^* = \frac{4i\omega k}{c} \int S \left( C_{1x} G_{1x}^* d^2x \right) \left( G_{1x} \nabla G_{2x}^* - \nabla G_{1x} G_{2x}^* \right) dx,
\]

where \( G_{ij} \) is the (scalar) Green’s function associated with source location \( i \) and receiver location \( j \), \( c \) is the phase velocity, and \( k \) is the attenuation. Note that Eq. (1) has been demonstrated in a 3D complex medium (Snieder et al., 2008; Campillo and Roux, 2014) and is applied here in a 2D configuration. After Fourier transformation back to the time domain, the two terms on the left-hand side of Eq. (1) are the “causal” and “anti-causal” parts of the Green’s function; i.e., the Green’s function between a source at the location of receiver 1, recorded at receiver 2, and the reciprocal Green’s function (a source at receiver 2, recorded at 1), respectively. \( S \) is the surface occupied by the sources, and \( \Gamma \) the boundary of this surface (Wapenaar, 2004). In Eq. (1), the surface integral corresponds to a homogeneous spatial distribution of sources inside the contour \( \Gamma \). The other integral can be interpreted as the intensity flux through the contour \( \Gamma \). In the case where the contour is in the far-field of the receivers and the medium heterogeneities, the line integral can be formulated as the correlation performed from sources at every point along the contour \( \Gamma \) (Wapenaar, 2004). In practice, the spatial source averaging of cross-correlations is the processing strategy (usually known as stacking) that is followed in seismological applications, and it is the strategy also chosen here, to evaluate the Green’s function retrieval of reverberated echoes.
Many sources (or alternatively, many scattering obstacles) are needed for the wave-field to become diffuse, which is essential for the Green’s function to be retrieved from a stack of cross-correlations. In practice, if the complexities of the medium (heterogeneities, interfaces, or scatterers) are confined within $\Gamma$ and attenuation $\kappa$ is neglected, a simple layout of point sources, such as a circle or an ellipse enclosing the receivers, might be sufficient to reproduce $G_{12} - G_{21}$ (Snieder, 2004; Froment et al., 2010).

In this approach, the surface integral in Eq. (1) can be neglected. If the interest is simply in the reconstruction of the “first arrival,” i.e., the direct path of the Green’s function, it might be enough to have sources in the so-called “stationary-phase regions,” i.e., aligned with the azimuth defined by the receiver pair (Fan and Snieder, 2009; Roux et al., 2005; Curtis et al., 2012). If the medium is more complex, including scatterers, boundaries, and/or smooth heterogeneities, the Green’s function results from the superposition of a variety of arrivals, and it is harder to reconstruct. In this case, it becomes necessary to illuminate the medium with a uniform, dense distribution of sources, to achieve the complete reconstruction of the Green’s function (Sato, 2010).

In the literature, numerical and laboratory experiments have described the Green’s function recovery in three dimensions through correlation (Snieder and Fleury, 2010; Fleury et al., 2010; Mikesell et al., 2012) with sources located along a surface that surrounds the receiver pair, but never in the volume of the sample. In two dimensions, this would be equivalent to neglecting the surface integral on the right-hand side of Eq. (1). Using a simple 2D numerical example, it is demonstrated how the presence of only one reflecting boundary, responsible for a second arrival in the Green’s function, requires the source distribution to be uniform over the surface for the Green’s function to be accurately reconstructed. Figures 1(a) and 1(b) show the uniform and circular source layouts, respectively. The receivers,
respectively R1 and R2, are 60 cm away from one another, while the boundary is at a distance of 3 m from both R1 and R2, in the “north” direction. In this first numerical experiment, the wave propagation is modeled via a simple ray-theory approximation; i.e., a Gaussian pulse centered at 18 kHz that propagates along a straight ray path with an amplitude weighted according to the geometrical spreading.

The cross-correlation patterns in Figs. 1(c) and 1(d) show the arrival times of different wave packets that depend on the azimuth of the source with respect to the two receiver positions. Each cross-correlation in the gathers corresponds to an average cross-correlation over a set of sources that are grouped in 0.5° bins, according to their azimuth θ. The cross-correlation patterns are dominated by the “ballistic” term that corresponds to the direct wave recorded at the two receivers, and for which the arrival time in the cross-correlation changes smoothly with the azimuth. The stack in Fig. 1(e) results from the uniform source distribution of Fig. 1(c). As expected from the surface integral in Eq. (1), this reveals the “direct” arrival, which propagates from one receiver to the other, and the “reflected” arrival, which propagates from one receiver to the boundary and, after reflection, to the other receiver. These two wavelets result from constructive interference due to sources located in the stationary-phase areas (Roux et al., 2005; Snieder, 2004) that are clearly visible in both the causal (positive-time) and anti-causal (negative-time) cross-correlations, at ±0.7 and ±6 ms, respectively. The wavelet associated with the reflected wave in the cross-correlation gather in Fig. 1(c) is characterized by a sudden amplitude drop outside the stationary-phase region [marked by arrows in Fig. 1(c)]. On the other hand, Fig. 1(d) shows that a number of stationary-phase points also occur with the circular source distribution, which in the stacked cross-correlation [Fig. 1(f)] gives rise to a set of wavelets where their arrival times depend on the radius r1 of each source circle. These arrivals are not part of the Green’s function associated with the receiver locations, which depends only on the medium properties and the interface geometry. These wavelets are called “spurious” (nonphysical). Increasing the distance of the sources with respect to the receiver separation is not sufficient to remove the spurious arrivals. Note that the total number of sources used is similar in Figs. 1(a) and 1(b), which means that the spurious arrivals do not depend on the sampling of the circular patterns. If the exact Green’s function is to be retrieved by interferometry, these spurious arrivals have to be eliminated by improving the source distribution. In Fig. 1(c), no stationary-phase points are observed, other than those associated with the arrivals in the true Green’s function, and the stacked cross-correlation [Fig. 1(e)] consequently does not include any spurious wavelets. Therefore spurious arrivals are cancelled more efficiently by a spatially, rather than azimuthally, uniform source distribution.

With the simple example of a two-wave problem (direct and reflected), this numerical experiment demonstrates the contributions from the cross terms in the cross-correlation that leads to spurious arrivals. This is a general feature for multi-wave problems, as classically encountered in elastic media with longitudinal P waves and transversal S waves. Similar arguments can be drawn that indicate the importance of the source–receiver spatial diversity. In the following, numerical simulations are used to further compare the spatially uniform and contour-like source distributions in more complex and realistic scenarios that include dispersion and multiple reverberations.

II. NUMERICAL SIMULATIONS OF FLEXURAL WAVES

Flexural waves generated in a thin 2D plate are a good example of dispersive waves (Larose et al., 2004). Propagation of A0 Lamb waves (Lamb, 1904) are numerically modeled using the spectral element method (SEM) (Padovani et al., 1994) in the 1.5 × 1.5 m thin plate (2 mm thickness) with free boundaries depicted in Fig. 8. The plate is made of aluminum, which is characterized by a density ρ = 2710 kg/m³, a compressional velocity Vp = 6300 m/s, and a shear velocity Vs = 3100 m/s. The plate is assumed to be isotropic and homogeneous.

The present approach differs from other studies where numerical simulation has been used (Snieder et al., 2008), in that it allows for a much longer propagation time within a closed, reverberating cavity; there is time for multiple border reflections, which results in a very complex field.

A. Simulation setup

To simulate the propagation of elastic waves on the plate, the latest release of SPECFEM3D (Peter et al., 2011) is used in combination with the meshing tool CUBIT. After decomposition of the mesh in a load-balanced fashion, the simulation is executed in parallel to calculate a ~25-ms-long accelerogram. In such a short time, a wave at 10 kHz bounces back and forth from the sides of the plate more than 20 times. As the computational cost of each simulation is considerable, it is possible to swap the roles played by the sources and receivers in the wave equation by using the reciprocity theorem (Aki and Richards, 1980; Curtis et al., 2012). To simulate a complex reverberated field, only two simulations need to be conducted, each initiated by a single source placed at the location of one of the receivers, R1 and R2. The signal is then “recorded” at source locations S, which in analogy with Sec. 1A above, are alternatively distributed uniformly over the surface of the plate [Fig. 2(a)], or along an ellipse that surrounds R1 and R2 [Fig. 2(b)]. This strategy does not affect the results, and it is common practice in this type of numerical experiment (Mikesell et al., 2012).

The source signal emitted respectively in R1 and R2 is a Gaussian time function that acts in the vertical direction and excites the 10–25 kHz frequency band (Komatitsch and Tromp, 2002). Note that the excited plate waves are dispersive A0 Lamb waves, which have a minimum phase velocity of ~600 m/s and a maximum of ~1400 m/s in this band. The receivers are located at the surface and record the z-component of the acceleration. Importantly, attenuation is neglected in the simulations. The external coupling of the plate with the air and with the supporting structure is likewise not modeled. Therefore, these numerical simulations are not expected to perfectly match experimental laboratory results.
For comparison purposes, the plate response is also computed between R1 and R2; i.e., the Green’s function that besides the first arrival, contains other secondary phases due to reverberation. Figure 3 describes the first wave packets as a set of reflections from the four edges of the plate. The main difficulty in this exercise is to isolate single phases when the medium is dispersive and reverberant. Due to dispersion, a series of wave packets are partially distinguishable after the direct arrival, each of which contains reflected phases. From the travel-time of the direct arrival, a group velocity of \( \frac{1200}{C_2} \) m/s is obtained that corresponds to the central frequency of the signal. Three other travel-times are derived easily from a geometric analysis of each source–boundary–receiver configuration, using the blueprint in Fig. 2. Any arrivals after these four phases result from interfering paths, because of multiple reflections at the plate boundaries.

B. Elliptical versus uniform distribution of sources

Figures 4–7 show the results of the averaging of the cross-correlations of the complex wave-fields from the source distributions of Figs. 2(a) and 2(b). The choice of an ellipse instead of a circle for the source distribution in Fig. 2(b) is motivated by the stack of cross-correlation functions with the same amount of dispersion. Indeed, the effect of dispersion (and attenuation whenever considered) classically depends on the path length. In the correlation process, it can be shown that the addition of the path lengths between each source(s) and the receivers in R1 and R2, [namely \( sR_1 + sR_2 \); see Fig. 2(b)] is independent of the source position along each ellipse with foci in R1 and R2 (Roux et al., 2005). The cross-correlation gather and the corresponding stack in Fig. 4 are obtained after the selection of a 2-ms time window starting from 0 time, which contains both the direct arrival and a few of the later arrivals, as discussed in Fig. 3. Note that prior to stacking, the cross-correlations are averaged over 0.5° bins according to the azimuth \( \theta \).

The cross-correlation pattern in Fig. 4(a) is qualitatively analogous to that of Fig. 1(c), and is dominated by the ballistic arrivals in each correlation function. The constructive interference that results in the direct arrival in the cross-correlation stack corresponds to the stationary-phase region at \( \approx 0.5 \) ms (Fan and Snieder, 2009). The stack matches relatively well the portion of the reference Green’s function that corresponds to the direct arrival, but the fit deteriorates before and after.

By cross-correlation over a longer time window (Fig. 5), the gather is no longer characterized by the ballistic signature, but rather by the constant-arrival-time stripes that can be associated to the predicted arrival times: (1) at \( \approx 0.5 \) ms, the direct phase from R1 to R2 and vice versa; and (2) at \( \approx 1 \) ms, the phases traveling from R1 (and, respectively, R2) to the two closest reflecting sides, and then, after

FIG. 2. (Color online) Blueprint of the plate showing the two receivers R1 and R2 and the two source distributions used during the numerical examples. The receivers are intentionally misaligned from the center of the plate to avoid symmetry. (a) The plate with the sources uniformly distributed over the surface at 0.06 m spacing. The azimuth (\( \theta \)) is computed according to the convention \([-\pi, \pi]\), and it grows counter-clockwise, as indicated by the arrow. (b) The plate with sources arranged along an elliptical contour surrounding the receivers. The dotted lines sR1 and sR2 indicate the source–receiver paths for a given source s. The same convention is used for the azimuth. The two distributions contain the same number of sources (\( \approx 600 \)) to avoid any bias in the comparison.

FIG. 3. (Color online) Temporal representation of the first 2 ms of the normalized Green’s function that contains direct arrivals, and the first reflections from the plate edges labeled after a travel-time analysis according the source–boundary–receiver distances.
reflection, to R2 (and, respectively, R1). Importantly, the stacked cross-correlation practically coincides with the reference Green’s function, except for 0 time, where (weak) spurious oscillations (i.e., non-physical wiggles generated by the cross-correlation of the border reflections) pollute the trace.

Some authors (Fan and Snieder, 2009; Wapenaar et al., 2010) use virtual or actual rings of sources to generate complex wave fields and to study the role of azimuthal source density on the stacked cross-correlations (Froment et al., 2010). Following this approach, Figs. 6 and 7 show the cross-correlation results with the source–receiver configuration of Fig. 2(b). The cross-correlations in Fig. 6 were computed over a short time window (2 ms), as in Fig. 4; however, the gather plot is much more complicated, with the superimposed ballistic signature of numerous phases other than the obvious, “direct” phase. By considering just the direct arrival plus the four boundary reflections, a total of 25 cross-correlated terms are expected, of which 10 are physical while 15 are due to spurious oscillations. In analogy with Fig. 5, the ballistic signature is lost by increasing the length of the cross-correlation window. The gather in Fig. 7(a) is characterized by causal and anti-causal arrivals that sum up constructively in the stack [Fig. 7(b)], which correspond to the causal and anti-causal direct waves and the edge reflections also seen in Fig. 5. However, vertical stripes of high

![Image](a)

![Image](b)

**FIG. 4.** (Color online) (a) Cross-correlations gather for the source distribution in Fig. 2(a) computed over a 2-ms time window, which includes only the first arrival and the secondary reflections. The traces are ordered according to the azimuth and have a frequency content between 10 and 25 kHz. The averaging procedure used in Fig. 1(d) defined over 0.5° bins has been applied here too. The signal coming from sources located within a wavelength (at 10 kHz) has been muted to avoid near-field effects. (b) Stacked cross-correlation (black) compared to the reference Green’s function (light gray).

![Image](a)

![Image](b)

**FIG. 5.** (Color online) Same as Fig. 4, but for a time window of 25 ms starting from 0 time.

![Image](a)

![Image](b)

**FIG. 6.** (Color online) (a) Cross-correlations gather for the elliptical source distribution in Fig. 2(b) computed over a 2-ms time window, which includes only the first arrival and the secondary reflections. The traces are ordered according to the azimuth and have a frequency content between 10 and 25 kHz. (b) Stacked cross-correlation (black) compared to the reference Green’s function (light gray). Note that the total number of sources remains the same as in Fig. 4.
III. LABORATORY EXPERIMENTS

In practical applications of the cross-correlation process, the spatial-temporal complexity of the noise source is one of the key factors to reconstruct the Green’s function. One limit of numerical simulation is the difficulty (if it is possible at all) of implementing an actual spatially diverse and temporally uncorrelated source signal (Peter et al., 2011). On the contrary, this can be achieved in the laboratory; e.g., by means of air-jet forcing (Larose et al., 2007). Compared to a deterministic point source that is typical of laboratory experiments (Mikesell et al., 2012), an air-jet flow is turbulent, which results in spatially and temporally random forcing in the target area.

A. Experimental setup

Flexural waves are excited on the $1.5 \times 1.5 \times 0.002$ m aluminum plate described in Fig. 8(a). The coupling between the plate and its supporting frame is damped by means of small absorbing-elastic patches that prevent resonance effects. Two broadband Brel & Kjaer accelerometers are attached at the center of the plate, at 60 cm apart and with a relative azimuth that is roughly parallel to one plate edge (Fig. 8). The accelerometers ensure a flat frequency response between 500 Hz and 15 kHz. The sampling frequency of both of the accelerometers is set at 100 kHz. Using either a piezoelectric transducer (around 10 kHz) or an air-jet generated by an air nozzle (around 2 kHz), $A_0$ Lamb waves (Lamb, 1904) are generated in the plate.

B. Excitation by air-jet sources

Acting at 10–20 cm above the plate, the air-jet produces a wide (tens of cm) turbulent air flow that creates a random-like pressure field at the plate surface. Moreover, the air-jet was also moved a few centimeters around a central position [see Fig. 8(a)], which further contributes to the creation of a continuous incoherent extended source. The total time of the continuous recording is about 250 s, which is divided into 5-s-long time windows, with each 5-s interval corresponding to a different position of the air nozzle, roughly along the inner ellipse in Fig. 8. The recorded signal is band-passed in the 0.5–5 kHz frequency range, and no further processing is applied (i.e., no whitening or “one-bit” filtering). Each 5-s-long continuous record is used in the same way as the point source signals presented in Sec. II. Following common practice in ambient noise seismology (Poli et al., 2012), the cross-correlations are computed over the largest available time window for which continuous recordings are available (i.e., 5 s), regardless of the expected duration of the Green’s function. The gather in Fig. 9, which is obtained by cross-correlation of each 5-s-long record, is quite different from that seen so far in this study. Unlike Figs. 1(c), 1(d), 4, and 6, no ballistic signature is visible, although maxima with constant arrival times appear, as in Figs. 5 and 7, where late arrivals were considered. These maxima can be associated to the direct ($\pm 1$ ms) and reflected arrivals of the Green’s function. We infer that a single realization of the experiment (a single cross-correlation in the gather) might be sufficient to generate a wave-field that is sufficiently spatially and temporally complex for the Green’s function to emerge.

The stack in Fig. 9 shows that the causal and anti-causal contributions of the correlation function are close to being symmetric. Symmetry with respect to time is an essential property of the averaged correlation function, and it is used here as a heuristic measure of the Green’s function retrieval. Indeed, measurement of the actual Green’s function would require a low-frequency piezo-transducer, which was not available at the time of the experiment. From Fig. 9, it appears that not only has the direct arrival of the Green’s function been extracted, but also that later arrivals ($\pm 3$ ms)
are visible. With an analysis analogous to that of Fig. 3, it is possible to attribute each peak to a boundary reflection. The results from this experiment highlight the robustness of the cross-correlation measurements when they are used with a spatially uncorrelated source and long time recordings.

C. Point-source results

In the point-source experiment, piezoelectric transducers are coupled with the plate at every position of the two ellipses shown in Fig. 8(a). A broadband Gaussian pulse centered at 10 kHz is used as the source function. After each pulse, a ~60-ms signal is recorded, as shown in Fig. 8(c), which is sufficient for flexural waves to propagate ~40 times back and forth in the 1.5-m-wide plate. The signal recorded was averaged over 100 realizations at the same location, to further improve the signal-to-noise ratio. The transducer is then shifted step-by-step along two contours of approximately elliptical shape [Fig. 8(a)], and the correlation process iterates over each source. As in the air-jet example, the symmetry of the causal and anti-causal contributions is used to benchmark the quality of the reconstructed Green’s function.

In Fig. 10, the cross-correlation is shown for a 3-ms time window that includes direct arrival and first border reflections only, while 40 ms of signals (i.e., including the late reverberation) are cross-correlated in Fig. 11. Using the shorter time window, the cross-correlation results (Fig. 10) are dominated by the ballistic arrival (and other not well identifiable patterns), as confirmed by the two early maxima that are visible in the stack [Fig. 10(b)]. Similar to other point-source results illustrated above with numerical simulations, the arrival times of the causal and anti-causal direct wave at ±0.7 ms correspond to the stationary phase regions in the gather. In this short window case, the stack is not symmetric. For the longer time window case, the stack is of greater quality, although spurious terms do not disappear completely, which leaves a residual asymmetry [Fig. 11(b)]. Later arrivals are not clearly visible in either the gather or the stack.

As expected, in the case of point-like sources, cross-correlations strongly depend on the azimuth of the source. A single shot from the transducer does not generate a wavefield that is diffuse enough, despite the numerous reverberations from the plate boundaries. The recovery of the direct arrival is only possible by stacking a large number of cross-correlations, which means that a sufficient source diversity (each cross-correlation in the gather) is necessary. The recovery of secondary arrivals appears to be difficult even with the use a long reverberation, as spurious oscillations do not cancel out, thus polluting the stacked correlation.
IV. CONCLUSIONS

Using flexural waves on a thin 2D plate, we proposed an analysis supported by laboratory and numerical experiments that aims to separate the relative roles of the reverberation and the source distribution that gives rise to a complex wave-field. A wave-field can be considered as “sufficiently diffuse” when the stacked cross-correlation of the recorded signals approximates well the Green's function, including phases other than the direct arrival.

The way the plate is excited is critical to understand the spatial-temporal complexity of the generated wave-field. For individual point sources, the ballistic signature is dominant in the cross-correlation gather when short time windows are considered, and the wave-field is clearly not sufficiently complex for the Green’s function to be adequately extracted by a single or a stack of a few cross-correlations. The Green’s function reconstruction is nearly complete only after a large number of averages, with the associated point sources uniformly covering the target area, and using a long time window for computing the cross-correlation that includes many reflections from the edges. In general, increasing the duration of the recorded time window, and hence including long reverberations, drastically improves the cross-correlation results for both the direct and secondary arrivals.

For the cross-correlation of continuous ambient noise to approximate well the Green’s function, it is also essential that a broad area is covered. It is shown experimentally that an air-jet exciting a finite area of an aluminum plate is much...
more effective than the combination of point sources in the reconstruction of the Green’s function from the correlation process. Such continuous incoherent extended source recalls the excitation of seismic noise by oceanic storms, which leads to great results for surface-wave extraction in seismology (Shapiro and Campillo, 2004; Ritzwoller et al., 2007).

This experimental demonstration is also demonstrated numerically, as a spatially uniform distribution of point-like sources is accordingly more effective than an azimuthally uniform source distribution along a close line, even if the total number of sources is the same. In its most general form, the correlation theorem (Wapenaar, 2004) [Eq. (1)] states that the 2D Green’s function associated with the location of two receivers is obtained by combining a surface and a line integral over the region populated by sources and its boundary. In the case of complex 2D media, the surface integral has a dominant role in the reconstruction of the complete Green’s function.

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