On the emergence of the Green’s function in the correlations of a diffuse field

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A diffuse acoustic field is shown to have correlations equal to the Green’s function of the body. Simple plausibility arguments for this assertion are followed by a more detailed proof. A careful version of the statement is found to include caveats in regard to how diffuse the field truly is, the spectrum of the diffuse field, and the phase of the receivers. Ultrasonic laboratory tests confirm the assertion. The main features of the direct signal between two transducers are indeed recovered by cross correlating their responses to a diffuse field generated by a third transducer. The quality of the recovery improves with increased averaging and the use of multiple sources. Applications are discussed. © 2001 Acoustical Society of America. [DOI: 10.1121/1.1417528]

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I. INTRODUCTION

The literature of the last several years on diffuse ultrasonic fields in solids,1–6 on diffuse vibrations in structural acoustics,6–11 and on diffuse seismic fields in the earth’s crust12–14 has largely neglected the field’s phase, and focused instead on the field’s spectral energy density. This is for good reason; fields which have reflected or scattered many times from specimen surfaces or inclusions resist detailed analysis. The spectral energy density of such a field evolves slowly and in a manner that may be comprehended by theory, but the phase is not predicted. The amplitude of typical diffuse field signals is predictable; the details of the stochastic waveforms are not. More recently it has been pointed out that diffuse field phase, while not predictable, varies with specimen temperature in a simple fashion.15,16

While phase is not predictable, in many systems it is coherent. There are definite correlations that survive the multiple scattering. These correlations manifest in striking phenomena, acoustic time reversal (e.g., Refs. 17–20), and weak localization21–23 being among them. Kramers–Kronig relations indicate that phase is related to features in spectral power densities, so a recent report that a diffuse field has fine spectral features related to local geometry24 implicitly also indicates the field’s phase has such information. There are many other cases in which the field is generated by a third transducer and is not necessarily fully diffuse and/or may have spectral features of its own. Theoretical estimates are suggested for the amount of averaging to achieve convergence may be achieved. An ultrasonic laboratory demonstration, for the case of a reverberant solid body, is presented in Sec. IV. The article concludes with a discussion of some remaining discrepancies and of potential extensions and applications.

II. PLAUSIBILITY ARGUMENTS

That the correlations of a diffuse field are related to local transient responses is easy to establish. In this section two distinct arguments are presented. The first is based on the standard definition of a diffuse field, as one with uncorrelated random modal amplitudes with equal variances. The second is based on an assumption that an instantaneous diffuse field is spatially uncorrelated.

A. Plausibility argument no. 1

A diffuse field $\phi$ in a finite body may be expressed in modal form by

$$
\phi(x,t) = \Re \sum_{n=1}^{\infty} a_n u_n(x) \exp\{i \omega_n t\},
$$

where the $a_n$ are complex modal amplitudes and the $u_n$ are the real orthogonal mode shapes. If the field is elastodynamic, $u$ and $\phi$ are vector valued. The $u_n$ are real and orthonormal:

$$
\int u_n \cdot u_m \, d^3x = \delta_{nm}.
$$

A statement that the field $\phi$ is diffuse with specified real spectral power density is equivalent to stating that the modal amplitudes are uncorrelated random variables

$$
\langle a_n a^*_m \rangle = \delta_{nm} \tilde{F}(\omega_n).
$$
where \( F \) is a smooth function related to spectral energy density, equal to \( \frac{1}{2}F(u^2) \) times the modal density.

It is a simple matter to construct the cross correlation of the fields at \( x \) and \( y \):

\[
\langle \phi(x,t) \phi(y,t+\tau) \rangle = \frac{1}{2} \mathcal{R} \sum_{n=1}^{\infty} F(\omega_n) u_n(x) u_n(y) \times \exp(-i\omega_n \tau). \tag{3}
\]

If \( F \sim \text{const} \) and an antiderivative \( \int_0^t \dot{F}(\tau) d\tau \) is taken, this is readily recognized as similar to the Green’s function \( G_{xy} \) governing propagation from \( x \) to \( y \):

\[
G_{xy}(\tau) = \sum_{n=1}^{\infty} u_n(x) u_n(y) \frac{\sin \omega_n \tau}{\omega_n} \quad \text{(for } \tau > 0, \text{ 0, otherwise).} \tag{4}
\]

The expression (4) differs from the time derivative of the actual Green’s function by (a) the presence of the factor \( F/2 \) which modifies its spectrum, and (b) by (3)’s support at negative \( \tau \).

### B. Plausibility argument no. 2

That local response would emerge from correlations of a diffuse field is also apparent from a propagator description. In the absence of external loads acting in the interval \([t, t + \tau]\), the field at time \( t + \tau \) may be determined by propagation from initial conditions at time \( t \) by means of a matrix multiplication:

\[
\begin{bmatrix} \phi(x) \\ \phi(y) \end{bmatrix}_{t + \tau} = \int \frac{1}{\rho(x)} \begin{bmatrix} G_{xy}(\tau) & G_{yx}(\tau) \\ \tilde{G}_{yx}(\tau) & \tilde{G}_{xy}(\tau) \end{bmatrix} \begin{bmatrix} \phi(x) \\ \phi(y) \end{bmatrix}_t \, d^3x. \tag{5}
\]

This expression is not widely employed, but it is not difficult to prove. It follows readily from the usual definition of the Green’s function as relating concentrated impulsive loads to subsequent responses, and a recognition that initial conditions \( \{\phi(x), \phi(x)\} \) may be established in a previously quiescent system by application of a distributed impulse \( \delta(x) \delta(t)/\rho(x) \) and a distributed derivative-of-an-impulse \( \delta'(x)/\rho(x) \).

A presumption that the fields at time \( t \) have no spatial correlations,

\[
\langle \phi(x,t) \phi(y,t) \rangle = \delta^3(x-y) \Phi(x); \quad \langle \phi(x,t) \phi(y,t) \rangle = 0,
\]

then leads to a conclusion that the fields at different times are correlated:

\[
\langle \phi(x,t+\tau) \phi(x,t) \rangle = \Phi(x)/\rho(x) \quad \tilde{G}_{xy}(\tau). \tag{7}
\]

An actual field can be expected to have short-range correlations at fixed times, correlations with a range inversely proportional to the bandwidth of the diffuse field. Thus this argument is incompletely satisfactory.

Each of the above arguments establishes the plausibility of the assertion. Each also indicates that there may exist subtleties related to the spectrum of the diffuse field. Each has conveniently disregarded other subtleties as to what precisely is meant by the averaging \( \langle \rangle \). In the next section we attempt a derivation of the central assertion, a derivation based on more precise statements about the spectrum of the diffuse field, and more precise statements as to what is meant by the averaging. It also treats practical complications that would attend the use of receivers with finite bandwidths.

### III. Detailed Argument

In practice, ultrasonic fields are diffuse only in a limited sense, even long after the source has acted. They do not precisely satisfy (2), if for no other reason then because it is far from clear what is meant by the averaging. The \( a_m \) are constants, independent of time and space. In practice there is no ensemble for laboratory averaging. Spatial averaging in the laboratory is avoided as tedious. Nevertheless, a particular transient source in a finite elastic body is expected to excite modes with no particular biases. There is a sense in which each mode receives a random amount of energy, proportional to its (random) participation at the site where the load is applied, and that short-range frequency averages ought to be equivalent to the theoretician’s ensemble averages.

Here we consider an ultrasonic field to be created by a deterministic transient load applied at a source point “\( s \)” in an elastic body. This is the case in laboratory practice. Subsequent to the action of this source, and subsequent to the ring time of the receivers, the responses in two receivers at \( x \) and \( y \), and at the source at \( s \), are given by

\[
V_s(t) = \sum_{n=1}^{\infty} u_n(x) u_n(s) \bar{X}(\omega_n) \bar{S}(\omega_n) \exp(i\omega_n t)/\omega_n,
\]

\[
V_x(t) = \sum_{n=1}^{\infty} u_n(x) u_n(s) \bar{Y}(\omega_n) \bar{S}(\omega_n) \exp(i\omega_n t)/\omega_n,
\]

\[
V_y(t) = \sum_{n=1}^{\infty} u_n(s) \bar{S}(\omega_n)^2 \exp(i\omega_n t)/\omega_n,
\]

where \( \bar{X} \), \( \bar{Y} \), and \( \bar{S} \) are the transducer receiver functions at \( x \) and \( y \) and the source function at \( s \). These expressions will be used only at late times, so the condition \( t > \text{ring time of transducers} \), need not be emphasized.

Our central assertion is that the direct signal from \( x \) to \( y \) is somehow present in the correlations of \( V_s \) and \( V_x \). But it will be seen that it is not the actual direct response that is recovered, but rather an acausal waveform \( u_{xy} \) given by

\[
u_{xy}(t) = \sum_{n=1}^{\infty} u_n(x) u_n(y) \bar{X}(\omega_n) \bar{Y}(\omega_n) \exp(i\omega_n t)/\omega_n \quad \forall t. \tag{9a}
\]

The acausal waveform \( u_{xy} \) (9a) is the signal that would be received if the Green’s function (4) were extended (acausally) to negative \( \tau \):

\[
u_{xy}(t) = \bar{Y}(t) \otimes \tilde{G}_{xy}^{\text{(Extended)}(t)} \otimes X(t), \tag{9b}
\]

where we have assumed that the source function \( X(t) \) of the transducer at \( x \) is identical to the receiver function of the same transducer. Equation (9b) differs from the actual signal \( v_{xy} \) received at \( x \) due to a source at \( y \):
\( u_{xy}(t) = Y(t) \otimes G_{xy}(t) \otimes X(t) \),

which is the convolution of transducer functions \( Y \) and \( X \) with the causal Green’s function (4). The two expressions are identical, however, at times after the ring time of the transducers \( X \) and \( Y \).

A temporal cross-correlation function between \( V_x \) and \( V_y \) is constructed by means of a time integration over a finite time interval \( \Delta T \) centered on time \( T \) with window function \( W(t) \):

\[
C_{xy}(\tau) = \int_{t = -\Delta T/2}^{t = \Delta T/2} W(t) V_x(T + t) V_y(T + t + \tau) \, dt.
\]

Averaging over an ensemble and/or over a set of source positions is also a possibility. On substituting (8) into (9), one obtains (after dropping the terms \( -\exp \{\pm i(\omega_n + \omega_m)t\} \) that vanish if the integration time \( \Delta T \) is long enough compared to the period of a typical frequency)

\[
C_{xy}(\tau) = \frac{1}{2} \sum_n \sum_m u_n(x) u_m(y) \left[ u_n(s) u_m(s) \right] / \omega_n \omega_m \Re \tilde{X}(\omega_n) \tilde{S}^* (\omega_m) \tilde{S}^* (\omega_m) \times \exp \{-i \omega_n \tau\} \tilde{W} (\omega_n - \omega_m) \exp \{i T (\omega_n - \omega_m)\}.
\]

The double sum now reduces to a single sum, the cross terms \( n \neq m \) vanishing. This follows immediately if the window duration \( \Delta T \) is long compared to the “break time,” \( 2 \pi / \Delta \omega \). It also follows, at moderate \( \Delta T \), if we perform a further average over a large number of window center times \( T \). This is rather similar to the case \( \Delta T \) large. It also follows in a statistical sense when it is recognized that the factor \( u_n(s) u_m(s) \) is stochastic, with zero mean unless \( n = m \). An average over all source positions \( s \) cancels the cross terms, too, due to orthogonality of the modes. Similarly, an average over a small number of discrete sources \( s \) should accelerate the cancellations. Thus we approximate

\[
C_{xy}(\tau) \approx \tilde{W}(0) \sum_n \left[ \frac{u_n(s)}{2\omega_n} \right]^2 |\tilde{S}(\omega_n)|^2 \Re \tilde{X}(\omega_n) \tilde{S}^* (\omega_n) \times u_n(x) u_n(y) \exp \{-i \omega_n \tau\}.
\]

Alternatively,

\[
\hat{C}_{xy}(\tau) \approx \tilde{W}(0) \sum_n \left[ \frac{u_n(s)}{2\omega_n} \right]^2 |\tilde{S}(\omega_n)|^2 \Re \tilde{X}(\omega_n) \tilde{S}^* (\omega_n) \times u_n(x) u_n(y) \exp \{-i \omega_n \tau\} / \omega_n.
\]

A similar relation was recently given by Draeger and Fink (Ref. 20, p. 613) in terms of convolutions between time-reversed and non-time-reversed signals, i.e., cross-correlation functions. They stated that the convolution of the backscattering impulse response at a source point \( s \) with the time-reversed version of \( u_n \) is the same as the convolution of the time-reversed response at \( s \) to the response at \( s \), with the response at \( y \) to the source at \( s \). Their expression, as given, is impossible to evaluate, inasmuch as backscattering point source impulse responses are singular at all times while the source is acting. We do not doubt, however, that the expressions are closely related.

Except for the factor \( |S|^2 |u_n(s)|^2 / 2 \) and the factor \( Y^* \), Eq. (14) rather resembles the acausal waveform (9). If the first factor could be eliminated, or deconvolved, and if the factor \( Y^* \) could be replaced with \( Y \), it would be precisely \( u_{xy} \). As it stands, (14) represents a distorted version of that response.

The first factor, \( |S|^2 |u_n(s)|^2 / 2 \), in the summation is a positive quantity that depends on the source. It is the work done by the source on mode number \( n \). We recognize that it is stochastic, with uncorrelated rapid variations among neighboring modes. If \( \tau \) is much less than the break time \( T_{\text{break}} \), at which these variations may be resolved, and if \( u_n(s)^2 \) is uncorrelated with \( u_n(s) \) and \( u_n(y) \), then the factor may be replaced with its (short range) frequency average. We define a real positive quantity \( f(\omega_n) \) that depends on source position and source time function, but varies only slowly with frequency:

\[
f(\omega) = \frac{1}{2} \langle |u_n(s)|^2 \rangle |\tilde{S}(\omega)|^2.
\]

That such modal factors are uncorrelated in this manner was also assumed in a recent derivation of SEA-like formulas for mean square responses in complex systems.\textsuperscript{24} The brackets \( \langle \rangle \) now represent an average over modes \( n \) with frequencies \( \omega_n \) in the vicinity of \( \omega \). The quantity \( f(\omega) \) is the spectral density of work done by the source, divided by the modal density in the structure. If \( |S| \) is known, and the structure is reasonably simple in the vicinity of the source, then \( f \) may be estimated analytically.

Too commonly, in ultrasonics anyway, transducer functions are not known. Very commonly the source vicinity is too complicated to allow analytic estimation of \( \langle u(s)^2 \rangle \). We therefore propose to determine \( f \) by measurements of the autocorrelation function of \( V_s \), a quantity that is not difficult to obtain.

\[
C_{ss}(\tau) = \int_{t = -\Delta t}^{t = \Delta t} W(t) V_s(T + t) V_s(T + t + \tau) \, dt.
\]

The power spectral density of \( V_s \) is the Fourier transform of \( C_{ss} \) and is clearly

\[
\tilde{C}_{ss}(\omega) = \pi \frac{d N(\omega)}{d \omega} \tilde{W}(0) \langle u(s)^4 \rangle |\tilde{S}(\omega)|^4 / 4 \omega^2,
\]

where \( d N/d \omega \) is the structure’s modal density. We take \( \langle u^4 \rangle = E(u^2)^2 \) and conclude

\[
f(\omega) = \sqrt{\frac{\omega^2 \tilde{C}_{ss}(\omega)}{2 E \tilde{W}(0) \pi [d N/d \omega]}}.
\]

The identification \( \langle u^4 \rangle = E(u^2)^2 \), with \( E = 3 \), is accepted for generic structures at late times \( T \). It corrects \( C \) for the modal echo, an enhancement of the signal in the vicinity of the source. At earlier times \( T \) that enhancement is only two.

\( E \)’s transition from 2 to 3 takes place on a time scale of the
break time, \( T_B = 2\pi dN/d\omega \), \( T_B \) is frequency dependent. Thus we take \( E \) to have a \( T- \) and \( \omega \)-dependent value between 2 and 3, as described elsewhere.\(^{21-23} \) In any case, variations in \( E \) are relatively unimportant compared to the more severe variations usually found in \( C_{ss} \).

We now conclude with a statement that the cross-correlation function between \( V_x \) and \( V_y \), after differentiation with respect to time, and deconvolving the known function \( f \) and \( Y^*/Y \), is

\[
\frac{i\omega C_{xy}(\omega)}{f(\omega)} \cdot \frac{Y(\omega)}{Y^*(\omega)} = \frac{i\omega \hat{V}_x(\omega) \hat{V}_y^*(\omega)}{f(\omega)} \cdot \frac{Y(\omega)}{Y^*(\omega)} = \hat{u}_{xy}(\omega) = \hat{X}(\omega) \hat{Y}(\omega) \hat{C}_{xy}^{\text{Extended}}(\omega).
\]

(20)

After inverse Fourier transforming this is the acausal signal \( u_{xy}(t) \), Eq. (9).

Equation (20) has been derived using a modal expansion valid in finite systems for which the time and/or spatial averaging is finite. There is good reason [cf. the plausibility argument Eq. (5)] for thinking it valid for open infinite systems, but a proof thereof has not been given. This is an important avenue for future research.

IV. SPEED AND ACCURACY OF CONVERGENCE

A priori estimates of the degree of averaging necessary before the Green’s function actually emerges from the correlation are possible. The derivation above [see discussion following Eq. (12)] suggests that time averaging of the correlation function over a period comparable to the break time, \( T_{\text{break}} = 2\pi \) mean eigenfrequency spacing \( = 2\pi /\Delta \omega \), should suffice. In fact, the result is exact if the time integration is taken over the break time and if an additional average over all source positions is taken. Break times usually exceed signal durations and therefore averaging over that much time is generally going to be impractical. In unbounded bodies, the break time is infinite. Averaging over all source positions is clearly impossible; averaging over a few may prove impractical. Nevertheless, there is reason to think that a more moderate amount of averaging should suffice if the intent is to construct waveforms over short times \( \tau \) only.

In the propagator description (5) one sees that the signal at time \( t + \tau \) and position \( y \) is composed of an incoherent superposition of contributions from a large number of distant (distance \( c \tau \)) positions \( x \). The number of such positions \( n \) may be estimated by assigning a cubic (square in 2-D) half-wavelength to each independent position \( x \), and calculating a volume (area) that contributes at time-difference \( \tau \) of \( 4\pi c [c \tau]^2 /3\Delta \omega \) (or \( 2\pi c [c \tau] /\Delta f \) in 2-D) where \( \Delta f = f \) is the bandwidth of the Green’s function that is to be recovered. The number of such positions is \( N = 32\pi (f \tau)^2 /3 \) in 3-D and \( 8\pi f \tau \) in 2-D. Simple arguments assuming all such positions contribute equally, but that most contribute essentially noise and only one of them (the one at the position of the other transducer) is not noise, then allow one to estimate that the signal emerges from the noise after a number of averages greater than the number of positions \( N \). As an independent term in the average is contributed at different times \( t \) separated by \( 1/\Delta f \), we conclude that the correlation function needs to be assembled from a signal of duration \( \Delta T \gg N/\Delta f \). This corresponds to \( \Delta T \gg 32\pi (f \tau)^2 /3 \) in 3-D and \( \Delta T \gg 8\pi f \tau \) in 2-D. In a two-dimensional structure with a multi-branched dispersion relation, e.g., a thick plate, this estimate must be augmented by a factor equal to the number of branches that contribute at the frequency of interest.

V. PRACTICAL IMPLEMENTATION

We have attempted to reconstruct local responses from diffuse fields in the simple blocklike structure of approximate dimensions \( 80 \times 140 \times 200 \) mm\(^3\) used in an earlier report.\(^{23} \) It is illustrated in Fig. 1. The signals \( V_x \), \( V_y \), and \( V_z \) were obtained using broadband Valpey–Fisher pintransducers for which the greatest sensitivity lies in a frequency range from 0.1 to nearly 2 MHz. The transducers were oil-coupled. Transient signals were generated by a Metrotek ultrasonic pulser, applied to transducer \( s \). They were then amplified by 40 dB using battery-powered Panametrics preamplifiers, and digitized at 5 MSa/s. A 6.4-ms duration window of the transient signal (32,000 points) in each of the three receivers, \( x \), \( x \), and \( y \), was captured, repetition averaged, and passed to a PC. The capture was then repeated for windows centered on successively later times \( T_D \). A direct application of Eq. (20) is not appropriate. The latter part of the \( \sim 160 \)-ms signal is significantly weaker than the initial part of it, as absorption has attenuated it. A direct implementation of (20) would effectively discard the great amount of information present in the latter parts of the signal.

To use all the information we therefore construct the following average:

\[
\bar{X}(\omega) = \sum_{D=1}^{N_D} \frac{i\omega \hat{V}_x^D(\omega) \hat{V}_y^{D*}(\omega)}{f(\omega) \exp\left[-2\gamma(\omega)T_D\right]},
\]

(21)

where the sum is over a series of \( N_D (\sim 25) \) nonoverlapping successive “delays,” each of width \( \Delta T = 6.4 \) ms, each centered on a different time \( T_D \). It may be noted that we have not attempted to correct for the phase \( \tilde{Y}(\omega)/\tilde{Y}^*(\omega) \). The exponential factor in the denominator is intended to compensate for absorption. In each term the factors \( V(\omega) \) in the
denominator are constructed from the Fourier transforms of the signals $V(t)$ in the delayed time window centered on $T_D$.

$V_s$ and $V_y$ were captured simultaneously with a two-channel digitizer. If the capture were not simultaneous, small unavoidable fluctuations in temperature, it can be shown, will introduce uncontrolled time shifts between the contributions recovered at large delays $D$.

In (21) the factor $f$, and the decay rate $\gamma$, were constructed from the power spectral density observed in $V_s$ by fitting $|V_s^p(\omega)|^2/E(\omega,T_D)$ to an exponential time dependence: $f(\omega)\exp[-2\gamma(\omega)T_D]$. The $f$ and $\gamma$ that are constructed from this show some random fluctuations that are at least partly a result of the finite sampling. They are therefore smoothed by application of cubic splines. Modal density $dN/d\omega$ is taken from theory. In three dimensions it is essentially proportional to $\omega^2$.

The resulting $\chi$ is inverse Fourier transformed and compared with the direct pitch-catch signal $u_{xy}$. Figure 2 shows these two waveforms. Figure 2(a) shows the waveform recovered by this process; Fig. 2(b) shows the pitch-catch waveform obtained by pulsing the transducer at $x$ and capturing the resulting transient at $y$. The main features of the direct pitch catch waveform [Fig. 2(b)] are clearly present in the correlation function [Fig. 2(a)]. One observes the direct Rayleigh wave, which arrives at 7 $\mu$s, and the signal due to a reflection of a $P$-wave from the bottom at about 30 $\mu$s. The high-frequency feature at 25 $\mu$s is a Rayleigh wave that has reflected from a nearby edge. These plots confirm the present assertion; the Green’s function is present in the correlations of a diffuse field. There are significant differences between the plots, however, and the question arises as to their cause.

Theory suggested that convergence is most thorough when an amount of time of the order of the break time is cross correlated. In this sample the break time is generally much greater than the 160 ms of data that was used to construct the correlation. (The break time is 120 ms at 200 kHz, and scales quadratically with frequency above there.) Therefore, the lack of perfect correspondence is not unexpected. Laboratory estimates may be composed for the degree of fluctuation that might be removed if one were to average over more time. The plot in Fig. 2(a) shows, in the horizontal dotted line, the root mean square deviation of $\chi(t)$ from its average among the 20 delays, divided by .20. This level is an estimate of the apparent error; its magnitude suggests that the differences between the correlation function and the direct pitch catch signal would be reduced with more averaging.

One way to do more averaging is to capture more than 160 ms. Unfortunately, the signal above 700 kHz and beyond 160 ms contains substantial noise and is not useful; high frequencies are too rapidly absorbed. To improve the convergence, we therefore eliminated the signal above 700 kHz by digital filtering, and captured a total of 50 delays, over 320 ms. The result is shown in Fig. 3, where it is compared with the digitally filtered direct pitch-catch signal. The agreement is much better.

Nevertheless, discrepancies remain. In an attempt to decrease these discrepancies, and in light of the theoretical indications that averaging over source positions $s$ may be useful, the process is repeated for eight positions of the source $s$. The average of these (with a fixed new choice for positions $x$ and $y$) is shown in Fig. 4. Figure 4(a) shows the comparison
between 900 kHz low-passed waveforms; Fig. 4(b) shows the case without low-pass filtering. For both plots, the discrepancies are reduced compared with those of Fig. 3. Remaining discrepancies are perhaps attributable to the phase of the receiver \( Y \) and/or to continued insufficiency in the amount of averaging.

VI. CONCLUSIONS

There may be some applications of these ideas in structural vibrations. The source of noise [or at least the source’s power spectrum \( f(\omega) \), with corresponding information about the source time function and local environment] might be inferable from Eq. (21) by comparing the noise correlations \( \chi \) with directly obtained local responses \( v_{xy} \). Arguments similar to those advanced here have recently been employed\(^24 \) to predict diffuse field energy spectra at \( x \) due to a source at \( x \), i.e., \( |V_x(\omega)|^2 \), in terms of the spectrum at \( s \) due to the source at \( s \), \( |V_s(\omega)|^2 \), and the spectrum at \( x \) due to a source at \( x \). It was argued there that such an ability may lend itself to efficient numerical predictions of mean power.

These observations may also have some applications to seismology. Increasing recent evidence\(^12–14 \) that seismic coda consist of multiply scattered, fully diffuse seismic waves suggests that correlations of seismic coda could reveal information on local stratigraphy. The very long ring times of lunar coda are especially intriguing. Cross correlation of years of seismic noise, on the earth or on the moon, might reveal features of local geologic structures.

Applications to ultrasounds are less clear. It would appear that the methods developed here, using a distant source to generate the diffuse field, might be useful whenever ultrasonic waveforms are desired from places at which it is difficult to insert an ultrasonic source, but where a receiver can be placed. A particularly intriguing possibility is using ambient thermal fluctuations to generate the diffuse ultrasonic field. Virtues of such a diffuse field include its broad spectrum (up to several TeraHertz at room temperature), the absence of any need to generate it, and its well understood statistics. It is, though, quite weak. That idea is pursued in other reports.\(^27 \)

Another intriguing idea is the possibility of applications to wave chaos theory.\(^26,28–30 \) In a ray-chaotic structure one expects rays to ergodically fill the entire available phase space. In practice, however, a ray that is absorbed, or reaches its “break time,” before filling phase space is effectively not chaotic, and the field not fully diffuse. Ray paths which are almost stable (“scars” in the language of the community) can be expected to contribute to a correlation to an excessive degree if the source lies on that path. If the source does not lie on that path, but the two receivers do, one would expect the correlation to show excessively weak contributions from that path. Thus scars should distort the correlations predicted in this report. In particular one expects ray-chaotic structures (like the one used here) to generate better correspondence between correlations and direct pitch-catch signals than do highly symmetric and/or pseudo-integrable systems.

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