Seismic Tomography

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- Solution quality (resolution)
Model Resolution

two types of resolution:

1. physical resolution
   -> what is the smallest block size we can resolve?

2. mathematical resolution
   -> which model parameters are resolved?
Model Resolution

1. Physical resolution

Physical resolution is important for image sharpness.

- 54 pixels
- 108 pixels
- 408 pixels
- 1120 pixels

-> depends mainly on frequency of our signal
Model Resolution

2. Solution quality

Goal is to identify regions that are

- poorly resolved
- unresolved
- well to fairly-well resolved

\[\text{Goal is to identify regions that are poorly resolved, unresolved, well to fairly-well resolved.} \]

\[\text{Depends mainly on ray distribution.} \]
Model Resolution

Ray distribution (coverage) per inversion cell

Each ray distribution has a different effect on the solution quality of the corresponding cell or model parameter.

- well resolved: many rays; good crossing
- fairly well resolved: many rays; no crossing
- poorly resolved: few rays; no crossing

-> How can we assess the ray distribution within each cell.
Model Resolution

How to assess solution quality?

1. Mathematical tools (resolution estimates):
   - hit count, derivative weighted sum (DWS)
   - resolution matrix (diagonal element, spread, resolution contours, full row)

2. Test with synthetic data:
   - spikes, checkerboards
   - synthetic structure models
Model Resolution

1. Hit count
   - is digital, i.e. cell is hit by ray or not
   - no information on ray length nor direction of ray

   hit count is equal for both rays, but impact on ray coverage is different

2. Derivative weighted sum (DWS)
   - is total weighted ray length in cell (weighted sum of derivatives),
   - information on ray length is included but not direction of ray,
   - measures ray density, e.g. high if rays pass close to grid node.
Model Resolution

Example DWS: Yellowstone dataset

Earthquakes concentrate in the northwest; no earthquakes below 6 km depth beneath the caldera.

-> very heterogeneous ray distribution!
Model Resolution

Example DWS: Yellowstone dataset

- High values in the NW at all depths
- Lower values inside the caldera below 8 km depth

-> No information on which values denotes poor or good resolution!
Model Resolution

DWS for different model parameterizations

DWS depends on model parameterization!

Good tool to select appropriate model parameterization (look for homogeneous ray distribution).

Data from Switzerland

10 x 10 km  15 x 15 km  20 x 20 km

z = 8 km  z = 8 km  z = 8 km

z = 30 km  z = 30 km  z = 30 km

Derivative Weighed Sum (DWS)
Model Resolution

Resolution matrix $R$ is defined as

$$m^{est} = R m^{true}$$

$-> R$ is an operator that tells us how well our model reflects the true model.

$$R = G^{-g} G$$

with $G^{-g} = (G^T G + \Theta I)^{-1} G^T$

(damped least square solution)

properties of $R$

- $R$ is a $m \times m$ matrix. ($m$ is number of model parameters)
- Each row of $R$ describes the dependence of one model parameter on all other model parameters.
- Diagonal element of $R$ ranges between 0 - 1, with 0=no resolution and 1=perfect resolution.
Model Resolution

Example RDE: Yellowstone dataset

- RDE varies between 0.1 and 0.7.

- High values in the NW between 2 km and 8 km depth.

- Very low values inside the caldera
Model Resolution

RDE and damping

RDE depends on the chosen damping!

-> Impossible to define general threshold for well and poorly resolved region.
Model Resolution

Yellowstone dataset (combined DWS and RDE)

Regions that show high values of both, DWS and RDE, are likely well to fairly-well resolved.

-> Only indirect assessment of cross-firing.
Model Resolution

Each row of the resolution matrix contains information on how the solution of the corresponding model parameter depends on its neighbors.

3-D visualization of one row of the resolution matrix.

- little smearing
- little vertical smearing
- strong vertical and horizontal smearing

-> Good but not possible for all model parameters!
Model Resolution

Spread function of a model parameter $j$ is defined as

\( S_j = \log \left[ |S_j|^{-1} \sum_{k=1}^{m} \frac{S_{kj}}{S_j} D_{jk} \right] \)

- \( S_{kj} \) = elements of \( j \)th row of resolution matrix
- \( D_{kj} \) = distance between model parameter \( j \) and \( k \)

- Compresses information contained in the row of the resolution matrix into a single number.
- Low \( S_j \) = low smearing
- Sums all elements of the row of the resolution matrix weighted by the distance to corresponding diagonal element.
Model Resolution

Example spread function: Yellowstone dataset

- Which spread value is still acceptable? -> Spread functions depend also on damping!

- Possible to “calibrate” spread function with plots of resolution row.

spread function $S_j$
Model Resolution

Resolution contours


- Contour row of resolution matrix in 2D.

- Define contour by threshold below which values of the resolution row have been decayed, f.e. 70% of the value of the diagonal element.

- Shape and spatial extent of contour can be interpreted as a measure of spatial smearing.
Model Resolution

Example resolution contours:
Yellowstone dataset

Combination of spread function and resolution contours allows comprehensive analysis of ray coverage.
Model Resolution

Summary resolution estimates:

• There exists a wide range of tools (resolution estimates) to assess ray coverage.

• Each of the different tools has its strength but also its flaws.

• Compare different resolution estimates and look for areas of homogeneously high values.

• Absolute values are difficult to interpret.

• No information on how well observed anomalies are resolved.
Model Resolution

Solution quality assessment with synthetic data:

- Compute synthetic travel times for same (real) earthquake-station distribution as in your data set.

- Add Gaussian distributed noise with a standard deviation that reflects your observation (picking) error.

- Invert synthetic travel times with same parameters as real data.

- Compare recovered and original velocity models.

- Common synthetic velocity models used:
  - Checkerboards or spikes (e.g. Spakman & Nolet, 1988),
  - Models with known and synthetic structures (e.g. Haslinger et al., 1999; Diehl et al., 2009).
Model Resolution

Example checkerboard model:
Yellowstone dataset

Alternating high and low velocity anomalies of certain size;
every second layer has no anomalies to check for vertical smearing.

(Husen et al., 2003)
Model Resolution

Example checkerboard model: Yellowstone dataset

Checkerboard model is well recovered for Vp in the northern part of the model but amplitudes are reduced; in the Vp/Vs model only the northerwestern part is well recovered with strong vertical smearing.

(Husen et al., 2003)
Model Resolution

Checkerboard model vs. characteristic model

Checkerboard models present very unrealistic models compared to the structure of the real Earth

-> only information on the sensitivity.

Characteristic models represent size and amplitude (characteristics) of the observed anomalies but with different geometry and/or sign

-> good models to assess solution quality and damping.

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Model Resolution

Characteristic model (for Alpine region)

Represents the characteristics of expected anomalies:

- Smaller and more complicated structures in the upper crust, reflecting the complex geology of the Alpine region.

- Larger and more simple structures in the lower crust, reflecting the variation in Moho depth beneath the Alpine region.
Model Resolution

Characteristic model (recovered)

Shape and amplitudes are well recovered.

Leakage problems

Heterogeneous RDE distribution

Homogeneous and relatively high RDE

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Model Resolution

Characteristic model
(calibrating RDE)

Shape and amplitudes with regions outlined by RDE=0.15 are well recovered; smearing can occur.

RDE=0.15 outlines well and fairly-well resolved regions.
Model Resolution

Special synthetic models to answer specific questions

No velocity smearing in the lower crust due to defocussing effects.

Velocity smearing in the lower crust due to focussing effects.

Very different consequences for interpretation!

(Diehl et al., 2009)
Model Resolution

Special synthetic models to answer specific questions

Results from Costa Rica.

Questions to answer:

1. Is the subducting slab a continuous feature?

2. Thickening of the low velocity anomaly real?

(Arroyo et al., 2008)
Model Resolution

Special synthetic models to answer specific questions

Synthetic slab model for Costa Rica (Arroyo et al., 2008):

- velocity perturbation: +/- 15 %
- continuous slab
- thickened low-velocity layer
Model Resolution

Special synthetic models to answer specific questions

Comparing input and recovered model

- Slab would be imaged as a continuous high-velocity feature
- Thickening of low-velocity layer is recovered, in particular at greater depth.

(Arroyo et al., 2008)
Model Resolution

Summary:

• Purpose of solution quality assessment is to outline regions, where the tomographic model is reliable.

• Resolution estimates only cannot achieve this purpose as they depend on a number of parameters (e.g. model parameterization, damping). Calibration with tests of synthetic model is needed.

• Tests with synthetic models are very useful to assess solution quality. They need to be designed, however, with care and with specific questions in mind.
Model Resolution

Presentation of tomographic results to reflect results from solution quality assessment.

Anomalies are reliable but smearing effects can be present. Should be interpreted very carefully or not at all.

(CSS Moho after Waldhauser et al. 1998)

(Vp=7.25 km/s Tomography Moho)

(Diehl et al., 2009)