Seismic Tomography

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- Solution of the forward problem
Solution of the forward problem

In order to compute the travel time residuals and partial derivatives we need to solve the, so called, forward problem.

travel time residual:
\[ t_i = t_0 + t_i^{\text{obs}} - t_i^{\text{calc}} \]

partial derivatives:
\[ t_i = \sum_{n=1}^{4} \frac{\partial f(h_n,m_k)}{\partial h_n} \Delta h_n + \sum_{k=1}^{k_{\text{tot}}} \frac{\partial f(h_n,m_k)}{m_k} \Delta m_k + e \]
Solution of the forward problem

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requirements:
- precise and reliable in 3D
- fast and efficient
Solution of the forward problem

Two main approaches exist:

**Ray Tracing**
- first approaches (1970s)
- bending, shooting, perturbation
- fast and efficient
- convergence problems, i.e. local minimum

**Grid Based**
- in the 1990s
- finite-difference (FD), graph theory
- computationally intensive
- always finds local minimum
### Solution of the forward problem

Three examples will be discussed in class:

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<th>Approximate ray tracing &amp; pseudo bending</th>
<th>3D ray shooting</th>
<th>3D-FD fat ray</th>
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<td>- implemented in SIMULPS</td>
<td>- implemented in SIMULPS14</td>
<td>- implemented in FATOMO</td>
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<td>- Fermat’s Principle</td>
<td>- Hamiltonian formulation of ray equations</td>
<td>- approximation of wave’s Fresnel volume</td>
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<td>- approximate, accurate up to 60 km ray length</td>
<td>- full 3D ray tracer</td>
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Solution of the forward problem

ART plus Pseudo-Bending
(Thurber, 1983; Um & Thurber, 1987)

1) approximate ray tracing (ART)

2) pseudo-bending

(piecewise adjustments of ray segments)
Solution of the forward problem

ART plus Pseudo-Bending
(Thurber, 1983; Um & Thurber, 1987)

1) approximate ray tracing (ART)

2) pseudo-bending

Will always give a ray path but maybe not the fastest one!

(piecewise adjustments of ray segments)
Solution of the forward problem

3D Shooting (RKP-ray tracing)
(Virieux and Farra, 1991; Haslinger, 2001)

1) Take initial take-off angles from ART-PB to find initial (central) ray that satisfies the ray equations

2) Perturb central ray by linear perturbation of the ray equations

3) Adjust initial angles of central ray according to paraxial ray endpoints

4) Repeat steps 2 and 3 if ray endpoint is not close enough
Solution of the forward problem

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Ray tracing can fail for very heterogenous models and long ray paths!
Solution of the forward problem

3D Shooting (RKP-ray tracing)
(Virieux and Farra, 1991; Haslinger, 2001)

Criteria for a successful ray:

1. Ray end point is within inner region

   - Take travel time as calculated,
   - Inner region should defined by accuracy of station coordinates (50 - 100 m),
   - For an accuracy of +/- 50 m and a near surface velocity of Vp=5 km/s we can expect a maximum error of 0.01 s.

(Haslinger, 2001; Maurer 2009)
Solution of the forward problem

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1. Ray end point is within inner region
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2. Ray end point is within outer region
   - Correct travel time by \( \Delta t = d \times \sin(\alpha)/Vp \),
   - Outer region should be defined in relation to the average reading error,
   - For a reading error of 0.05 s and a near-surface velocity Vp=5 km/s we get a maximum distance of 250 m.

(Haslinger, 2001; Maurer 2009)
Solution of the forward problem

Testing performance of ART-PB and RKP ray tracing by computing travel times from source to receiver and receiver to source

Synthetic model and source and receiver distribution (ray length from 5 km to 90 km).
Solution of the forward problem

Travel time differences between forward (source to receiver) and backward (receiver to source) for synthetic model:

(Haslinger & Kissling, 2001)
Solution of the forward problem

Travel time differences between forward (source to receiver) and backward (receiver to source) for synthetic model:

ART-PB and RKP are very accurate up to 60 km ray length.

(Haslinger & Kissling, 2001)
Solution of the forward problem

Travel time differences between forward (source to receiver) and backward (receiver to source) for synthetic model:

ART-PB and RKP are very accurate up to 60 km ray length.

RKP outperforms ART-PB for ray lengths > 60 km.

(Haslinger & Kissling, 2001)
Solution of the forward problem

Differences in ray paths:

(Haslinger & Kissling, 2001)
Solution of the forward problem

Differences in ray paths:

RKP rays are more realistic (bend around low velocity anomalies).

(Haslinger & Kissling, 2001)
Solution of the forward problem

Differences in tomographic results:

Solution (Vp)

Solution Quality (diagonal element of resolution matrix)

(Daslinger & Kissling, 2001)
Solution of the forward problem

Differences in tomographic results:

Solution of the forward problem affects both, solution and solution quality!
Solution of the forward problem

Differences in tomographic results:

Solution of the forward problem affects both, solution and solution quality!

Very difficult to assess for real data -> need for tests with synthetic models!
Solution of the forward problem

Finite-Difference (FD) solution to eikonal equation
(Podvin & Lecomte, 1991)

In the ray approximation, wave propagation is described by the eikonal equation \((\nabla t(x)) = s(x)^2\).

Wave fronts are represented by travel time isochrones.

Eikonal equation only gives first arriving waves, no secondary waves!
Solution of the forward problem

Finite-Difference (FD) solution to eikonal equation

(Podvin & Lecomte, 1991)

Finite difference solution implies that travel time computations are local: Arrival time at a given grid point depends only on arrival times on its neighbors.
Solution of the forward problem

Finite-Difference (FD) solution to eikonal equation

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\[ t_p = t_N + [(hs)^2 - (t_N - t_M)^2]^{1/2} \]

![Diagram showing travel times and wave propagation](image)
Solution of the forward problem

Finite-Difference (FD) solution to eikonal equation
(Podvin & Lecomte, 1991)

Finite difference solution implies that travel time computations are local: Arrival time at a given grid point depends only on arrival times on its neighbors.

transmitted wave (2x):
\[ t_p = t_N + [(hs)^2 - (t_N - t_M)^2]^{1/2} \]

head wave (2x):
\[ t_p = t_N + hs_2 \]
Solution of the forward problem

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diffracted wave (1x):
\[ t_p = t_M + h\sqrt{2} \]
Solution of the forward problem

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Finite difference solution implies that travel time computations are local: Arrival time at a given grid point depends only on arrival times on its neighbors.

Transmitted wave (2x):
\[ t_p = t_N + \sqrt{(hs)^2 - (t_N - t_M)^2} \]

Head wave (2x):
\[ t_p = t_N + hs_2 \]

Diffracted wave (1x):
\[ t_p = t_M + hs\sqrt{2} \]

-> take fastest travel time
Solution of the forward problem

Finite-Difference (FD) solution to Eikonal Equation
(Podvin & Lecomte, 1991)

- First arrival times are computed to all points in the model.

- Precision of FD depends on model grid spacing.

- Local travel time computation assumes locally planar wavefronts. Wave fronts are not planar close to source, i.e. they are strongly curved:

  -> Initialization with finer grid spacing around source region.
Solution of the forward problem

Initialization procedure:

(Podvin & Lecomte, 1991)
Solution of the forward problem

Initialization procedure:
(Podvin & Lecomte, 1991)

Travel times are computed on a densely spaced grid in the source region. This can be repeated iteratively.
Solution of the forward problem

Precision of FD based methods:

Travel time differences between forward (source to receiver) and reverse (receiver to source) travel times:
Solution of the forward problem

FD example in layered model

h = 0.5 km

6 km/s
8 km/s
Solution of the forward problem

FD example in layered model

How to compute rays?
Solution of the forward problem

FD example in layered model

How to compute rays?
Solution of the forward problem

FD example in layered model

How to compute rays?

Backward ray-tracing from receiver to source by following steepest gradient of the travel time field (a posterori ray tracing).
Solution of the forward problem

Fat rays approximating Fresnel volume
(Husen and Kissling, 2001)

Summation of source and receiver travel times fields:

\[ t_{sr} = \text{traveltime source to receiver} \]
\[ t_{sx} = \text{traveltime source to point } x, \]
\[ t_{rx} = \text{traveltime receiver to point } x \]
Solution of the forward problem

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Traveltime fields are computed using FD

\[ t_{sx} + t_{rx} - t_{sr} = 0 \]
Solution of the forward problem

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\[ t_{sx} + t_{rx} - t_{sr} = 0 \quad \text{-> ray} \]
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\[ t_{sx} + t_{rx} - t_{sr} = 0 \quad \rightarrow \text{ray} \]
\[ t_{sx} + t_{rx} - t_{sr} \leq T/2 \ (dt) \]
Solution of the forward problem

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\[ t_{sx} + t_{rx} - t_{sr} = 0 \quad \rightarrow \text{ray} \]
\[ t_{sx} + t_{rx} - t_{sr} \leq T/2 (dt) \quad \rightarrow \text{Fresnel volume} \]
(Cerveny & Soares, 1992)
Solution of the forward problem

Fat rays approximating Fresnel volume
(Husen and Kissling, 2001)

2D continuous:
(same 2 layer model as before)
Solution of the forward problem

Fat rays approximating Fresnel volume
(Husen and Kissling, 2001)

3D discrete (with blocks):

(3D synthetic velocity as used for ART-PB and RKP study)
Solution of the forward problem

FATOMO: Fat ray tomography
(Husen and Kissling, 2001)

ray tomography:
partial derivative = travel time along ray segment within inversion cell
Solution of the forward problem

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(Husen and Kissling, 2001)

ray tomography:
partial derivative = travel time along ray segment within inversion cell

fat ray tomography:
partial derivative = travel time scaled by volume of fat ray within inversion cell
Solution of the forward problem

FATOMO: Fat ray tomography
(Husen and Kissling, 2001)

ray tomography:
partial derivative = travel time along ray segment within inversion cell

fat ray tomography:
partial derivative = travel time scaled by volume of fat ray within inversion cell

In fat ray tomography travel time residuals are “projected” into the model in terms of volumes -> physical “smearing”
Solution of the forward problem

Comparing fat ray and ray tomography
(Husen and Kissling, 2001)

Real source-receiver distribution (Chile)

coarse grid spacing (20 km)

grid spacing >> fat ray width
Solution of the forward problem

Comparing fat ray and ray tomography
(Husen and Kissling, 2001)

Real source-receiver distribution (Chile)

coarse grid spacing (20 km)

Input

Fat ray

ART-PB

Similar performance of fat ray and ray tomography.
Solution of the forward problem

Fat rays approximating Fresnel volume
(Husen and Kissling, 2001)

grid spacing <= fat ray width

Real source-receiver distribution (Chile)
fine grid spacing (10 km)
Solution of the forward problem

Fat rays approximating Fresnel volume
(Husen and Kissling, 2001)

Real source-receiver distribution (Chile)

Fine grid spacing (10 km)

Better performance of fat ray tomography in fairly-well resolved regions!